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ON THE HALF LINE: K. RAMACHANDRA

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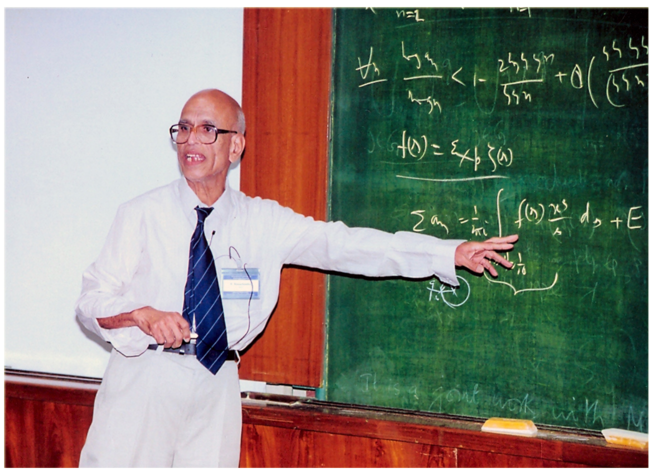

Abstract. This is a short biographical note[1] on the life and works of K. Ramachandra, one of the leading mathematicians in the field of analytic number theory in the second half of the twentieth century.

1. Introduction

Kanakanahalli Ramachandra (1933-2011) was perhaps the real successor of Srinivasa Ramanujan in contemporary Indian mathematics. Ramachandra has made invaluable contributions to algebraic number theory, transcendental number theory and the theory of the Riemann zeta function. This article is a brief exposition of the life and work of Ramachandra. The title of this biographical note, *On the half line*, is motivated by the fact that Ramachandra was one of the few mathematicians who was still working on the older classical problems in number theory and many of his best results are theorems related to the values of the Riemann zeta function on the half line, $\zeta(1/2 + it)$.

[1] This is a slightly revised version of the article that appeared in the September 2011 issue of the Mathematics Newsletter of the Ramanujan Mathematical Society.

2. Early life (1933-57)

Ramachandra was born on 18 August 1933 in Mandya in the state of Mysore (now known as Karnataka) in southern India. His grandfather walked nearly a hundred and fifty kilometers to see the new born Ramachandra. Ramachandra hailed from a family with a modest background; his father passed away when Ramachandra was only 13. Ramachandra's mother managed his education by taking a loan against their agricultural property. When Ramachandra was a student, he won a competition and was awarded a short biography of the legendary Indian mathematician Srinivasa Ramanujan. This was the book that ignited the interest for mathematics in Ramachandra.

Ramanujan's taxicab number $1729 = 9^3 + 10^3 = 1^3 + 12^3$ has become a part of mathematics folklore. During his college days, Ramachandra had a similar encounter with the number 3435. His college principal had a car with the number 3430 on the number plate. Ramachandra worked on the mathematical possibilities of this number and in the process found that upon adding 5, the number 3435 is a number with a distinct property that when each digit was raised to a power equal to itself and the resulting number was added up, the sum equals the original number i.e.

$$3^3 + 4^4 + 3^3 + 5^5 = 3435.$$

Ramachandra completed his graduation and post graduation from Central College, Bangalore. Due to family responsibilities, he had to look for a job at an young age; and just like Ramanujan, Ramachandra also worked as a clerk. Ramachandra worked as a clerk in the Minerva Mills where Ramachandra's father had also worked. In spite of taking up a job so remote from mathematics, Ramachandra studied number theory all by himself in his free time, especially the works of Ramanujan. Later, he worked as a lecturer in BMS College of Engineering. Ramachandra also served a very short stint of only six days as a research scholar in the Indian Institute of Science, Bangalore.

After the death of Ramanujan, Hardy wrote a series of lectures on the works of Ramanujan. These lectures were published as a book *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work.* This became one of the classic books on Ramanujan. When Ramachandra was a college student, he found a copy of this book in a public library in Mysore and he studied the works of Ramanujan with great devotion (unfortunately, someone had torn the cover of the book and taken away the picture of Ramanujan; this made Ramachandra very disappointed and, half a century later he still couldn't forgive the irresponsible person who tore the particular page). It was this book that inspired Ramachandra to become a mathematician and in particular, a number theorist. Later Ramachandra got a copy of the book which he always kept with him as a personal favorite.

3. TIFR Bombay (1958-1995)

In 1958, Ramachandra secured a research scholarship in TIFR and it was here that he met K. Chandrashekharan who was one of the experts in the theory of the Riemann zeta function in India at that time. Ramachandra studied the theory of the Riemann zeta function under K. Chandrashekharan. Later, Ramachandra himself was to become one of the leading experts on this topic and he made several invaluable contributions to the subject. For the next nearly four decades until his retirement, Ramachandra remained at TIFR and established one of the most prestigious schools of analytic number theory in collaboration with his gifted students as well as leading number theorists from all over the world.

Ramachandra believed that a mathematician must not only contribute to the subject but also play a role in guiding the next generation of mathematicians. He worked hard to perpetuate Number theory as an active research area in India and succeeded in inspiring interested students to take up the subject. Ramachandra acted as the doctoral advisor for eight students; today some of his students are among the most renowned mathematicians in the field of analytic number theory.

At the invitation of Norwegian mathematician Atle Selberg, Ramachandra went to the Institute of Advanced Study in Princeton, USA, as a visiting professor and spent a period of six months. This was Ramachandra's first foreign trip and years later when Ramachandra constructed his house in Bangalore, he named it 'Selberg House' in honour of Atle Selberg. Over the course of his career, Ramachandra visited several countries and collaborated with some of the leading number theorists and also invited many of the leading mathematicians to TIFR, including the legendary Paul Erdős. Erdős visited India in 1976 and stayed as a guest in Ramachandra's house. Ramachandra is one of the few mathematicians with Erdős Number 1. He published two joint papers with Erdős.

In 1978 Ramachandra founded the Hardy-Ramanujan Journal which, in its past issues, has carried some of the important contributions to number theory. It is one of the very few privately run mathematical journals in the world, funded entirely by Ramachandra and R. Balasubramanian who acted as its editors until the passing away Ramachandra in early 2011. The journal is published annually on 22 December, the birthday of Srinivasa Ramanujan.

4. NIAS (1995-2011)

After retiring form TIFR, Ramachandra returned to his home town Bangalore and joined the National Institute of Advanced Studies (NIAS) as a Visiting Professor on the invitation of nuclear physicist and Founder Director of NIAS, Dr. Raja Ramanna. Ramachandra remained in NIAS and continued working on the theory of the Riemann zeta function until his death. During this time he was also associated with TIFR Bangalore as a visiting faculty. In 2003, a conference was held on the occasion of the seventieth birthday of Ramachandra in TIFR Bangalore. Several mathematicians from all over the world attended the conference and celebrated the event.

Ramachandra *left* (in the words of Erdős) on 17 January 2011. His health had broken down and he had been hospitalized for about two weeks. According to his wife Mrs. Saraswati Ramachandra: *The doctors had advised him to take complete rest and not work on mathematics. But soon as he was discharged from the hospital, he started working on a problem that had been bothering him for the past six months.* He is survived by his wife and daughter.

5. Ramachandra's mathematical Gods

Although Ramachandra was born in a brahmin family and was a devote Hindu, he also had his own perception of Gods, his mathematical Gods. He used to address the great mathematicians as Gods and his reason was simple. *'Only a God can prove such a result,'* he used to say. Decorating the walls of his room in NIAS were the poster size pictures of G. H. Hardy, Srinivasa Ramanujan and I. M. Vinogradov; everyday as he entered his office he would put flower garlands on these portraits. *'They are my Gods'* he used to say as he spoke about them. Sometimes when he spoke about a particular mathematical result he used to thank his mathematical God who was an expert in that field. *'With Siegel's blessings, I was able to prove some results,'* Ramachandra said once, referring to the German mathematician C. L. Siegel, as he spoke about his theorems in transcendental number theory.

Once when I asked Ramachandra whom he thought was the greatest mathematician, *Ivan Matveyevich Vinogradov* he immediately replied. Vinogradov had devised an ingenious method of evaluating exponential sums which reduced the error in the prime number theorem to

$$\pi(x) = \mathrm{li}\,(x) + O(x \exp(-c_1 (\ln x)^{3/5} (\ln \ln x)^{-1/5})).$$

In the eighty years since Vinogradov published this result, no one was able to improve on it. Therefore Ramachandra held Vinogradov in the highest regard. In second position he named Ramanujan and in third position he named Hardy. Ramachandra visited Vinogradov twice when he was invited to attend the conferences held in Russia on the occasions of the eightieth and the ninetieth birthdays of I. M. Vinogradov.

6. Ramachandra as a teacher

Ramachandra has mentored some of the leading experts in number theory today including R. Balasubramanian and T. N. Shorey. He was extremely generous to his students and he often credited them in his papers even for very minor contributions.

Ramachandra was proud of the achievements of his students and often spoke about their works. *Balu and Shorey have brought a great name to me*, he said, referring to R. Balasubramanian and T. N. Shorey. It is often said in lighter vein that Ramachandra's greatest contribution to the theory of transcendental numbers is T. N. Shorey because Shorey has proved some very significant results in this field. During the days when R. Balasubramanian was his doctoral student at TIFR Bombay, Ramachandra often introduced him as: *This is R. Balasubramanian. We are teacher and student but at different times the roles will be reversed.*

7. Mathematical works

During his career as a mathematician, Ramachandra published over hundred and fifty papers which included several important works in the fields of algebraic number theory, transcendental number theory and the theory of the Riemann zeta function. Ramachandra was among the pioneers in evaluating the fractional moments of the Riemann zeta function. He was also the first mathematician to consider the gap between numbers with large prime factors. Several key areas of analytic number theory that Ramachandra has pioneered, continue to be active areas of research even today.

In his early years as a number theorist, Ramachandra worked in the field of algebraic number theory. His first paper was: Some applications of Kronecker's limit formula, Ann. of Math (2), 80(1964), 104-148. The reviewer M. Eichler remarked: *This paper contains some remarkable new results on the construction of the ray class field of an imaginary quadratic number field.* Ramachandra completed his PhD under the supervision of K. G. Ramanathan at TIFR Bombay (now known as Mumbai) in 1965.

When the seminal work of Alan Baker appeared in the 1960's, Ramachandra and his students, especially T. N. Shorey, took up transcendental number theory and made remarkable contributions to both the theory and its applications to problems of classical number theory. A detailed exposition of Ramachandra's contribution can be found in Michel Waldschmidt's paper 'On Ramachandra's Contributions to Transcendental Number Theory'.

In 1974, Ramachandra left transcendental number theory and turned his attention to classical analytic number theory, especially the theory of the Riemann zeta function and general Dirichlet series. His contributions to the theory of the Riemann zeta function is best summarized in the words of the British mathematician and a Fellow of the Royal Society, Roger Heath-Brown:

As soon as I entered research, 30 years ago, yours became a familiar name; and your influence has remained with me ever since. Time permits me to mention in detail only one strand of your work but it is one that clearly demonstrates how *important your research has been. A little over 20 years back you proved the first results on fractional moments of the Riemann Zeta-function. At first I could not believe they were correct!! Since then however the ideas have been extended in a number of ways. They have lead of course to a range of important new results about the Zeta-function and other Dirichlet series. But just as significantly the ideas have led to new conjectures on the moments of the Riemann Zeta-function. These conjectures provide the first successful test for the application of Random Matrix Theory in this area. Nowadays this is a growing area which has contributed much to our understanding of zeta-functions. And it can all be traced back to your work in the late 1970s.*

8. Selected theorems - The little flowers

Ramachandra used to dedicate some of his results to his mathematical Gods in papers whose title began with *Little flowers to ....* For instance when he visited Russia on the occasion of the ninetieth birthday of I. M. Vinogradov, the paper that he presented in the conference was titled *Little flowers to I. M. Vinogradov.* We shall present a few flowers from Ramachandra's garden that roughly cover his genre of work. For more details on Ramachandra's work, the reader is requested to consult the volumes of the Hardy-Ramanujan journal which are available online at http://www.imsc.res.in

Theorem 8.1. *(K. Ramachandra). Let λ be any constant satisfying $1/2 < \lambda < 1$ and l a non-negative integer constant. Put $H = T^{\lambda}$. Then we have*

$$(\ln T)^{1/4+l} \ll \frac{1}{H}\int_T^{T+H} \left|\frac{d^l}{dt^l}\zeta(1/2+it)dt\right| \ll (\ln T)^{1/4+l}.$$

Theorem 8.2. *(R. Balasubramanian and K. Ramachandra). Let t be a fixed transcendental number and $x \geq 1, y \geq 1$ be integers. Let n be any integer such that $x \leq n < x+y$ for which 2^{t^n} defined as $\exp(t^n \ln 2)$ is algebraic. The number of such integers is $\leq (2y)^{1/2} + O(y^{1/4})$.*

Theorem 8.3. *(M. Jutila, K. Ramachandra and T. N. Shorey). Let $k > 2$ and $n_1 = n_1(k), n_2 = n_2(k), \ldots$ be the sequence of all positive integers which have at least one prime factor $> k$. Put $f(k) = \max(n_{i+1} - n_i)$ the maximum being taken over all $i > 1$. Then*

$$f(k) \ll \frac{k}{\ln k}\Big(\frac{\ln\ln\ln k}{\ln\ln k}\Big).$$

Theorem 8.4. *(K. Ramachandra). For all sufficiently large m, between m^2 and $(m+1)^2$, there is an n and a prime p dividing n such that $p > n^{1/2+1/11}$.*

Theorem 8.5. *(Ramachandra, Shorey and Tijdeman). There exists an absolute constant $c_2 > 0$ such that for $n \geq 3$ and $g = g(n) = [c_2(\frac{\ln n}{\ln \ln n})^3]$, it is possible to choose distinct primes $p_1, p_2, \ldots, p_g$ such that $p_i|(n+1)$ for $1 \leq i \leq g$.*

9. Acknowledgement

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References

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