Towards a proof of the Hodge conjecture, and cycle spaces in positive characteristic

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Abstract. An English abstract is suggested and should be descriptive enough by itself. Please do not include citations, footnotes or references to numbered equations, theorems, figures or tables in your abstract. Avoid complicated formulae or displayed equations.

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1. Introduction

First of all, we want to stress that this paper has been typeset with the wonderful system \(\TeX\) created by Don Knuth [Knu84], in the form later adapted by Leslie Lamport [Lam94], the current version of which is \(\LaTeX\) (older versions won’t work).

The present text serves as an example to show how the \(\texttt{epimath}\) style works (using the class file \(\texttt{epiarticle.cls}\) and the more specific style \(\texttt{epimath.sty}\)).

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All data preceding the Introduction (Abstract, Keywords, MSC classification, table of contents should be enclosed by \(\texttt{\begin{prelims}} \ldots \texttt{\end{prelims}}\). In case you’d like to change the keyword \texttt{Abstract} into something else, e.g. \texttt{Summary}, use \texttt{def\{abstractname\{Summary\}} just before entering \texttt{abstract\{\ldots\}}. Similarly with \texttt{def\{keywordsname\{\ldots\}} to be used before entering \texttt{\documentstyle\{\ldots\}}, or \texttt{def\{MSCclassname\{\ldots\}}.

We thank episciences.org for providing open access hosting of the electronic journal \textit{Hardy-Ramanujan Journal}
2. Preliminary results

The following basic property will be used systematically.

Property 2.1. ...

3. Fundamental results about motives

3.A. A new functor

We start with a few basic facts that will be useful in the sequel. The first paragraph in a section, subsection or subsubsection is normally not indented (as you may guess, these are introduced by typesetting \section{...}, \subsection{...}, \subsubsection{...} respectively). You can force indentation by putting \forceindent before. We did so here.

However, if you start a new paragraph in the course of a continued exposition, The new paragraph will appear to be indented, unless you specify \noindent before.

By default, all statement environments are set so as to use the same counter for theorems, propositions, lemmas, to avoid the possibly confusing situation where different types of statements get the same numbering. We recommend to respect this rule for any new environment to be created. You can e.g. specify \newtheorem{assertion}[theorem]{Assertion} to define the new environment “assertion”, that will use the same counter as “theorem”.

You may wish to use a vertical skip to indicate a new logical step, otherwise there will be only a new paragraph without any vertical skip. Here we use \smallskip.

Our main mathematical statement is

\textbf{Theorem 3.1.} Let $F$ be a fonctor from the category of algebraic schemes to the derived category of $A$-modules. (…)

\textit{Proof.} The proof is a bit long, so we first present a short sketch. Surprisingly, the cohomology calculations require the famous Rogers-Ramanujan identity

\[ G(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q; q)_{n}} = \frac{1}{(q; q^5)_{\infty}(q^4; q^5)_{\infty}} = 1 + q + q^2 + q^3 + 2q^4 + 2q^5 + 3q^6 + \cdots, \quad (3.1) \]

where $(\cdot; \cdot)_{n}$ denotes as usual the $q$-Pochhammer symbol

\[ (a; q)_{n} = \prod_{k=0}^{n-1} (1 - aq^k) = (1 - a)(1 - aq)(1 - aq^2) \cdots (1 - aq^{n-1}), \quad (a; q)_{0} = 1. \]

Another step uses the dilogarithm function

\[ \text{Li}_2(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^2}, \quad (3.2) \]
Our arguments involve some preliminary steps which we call “first case” and “sporadic cases”, as detailed below.

**Remark 3.2.** We recommend Theorems, Propositions and Lemmas to be typeset in italics, Remarks and Examples in roman characters.

3.A.a. **First case**
We proceed here by induction on dimension.

**Initialization.** Assume \( n = 1 \).

**Inductive step.** Assume the result known up to dimension \( n - 1 \) with \( n \geq 2 \).

3.A.b. **Sporadic cases**
We settle the sporadic cases of Theorem 3.1 by means of Formulae (3.1) and (3.2).

3.B. **More about \( l \)-adic technology**
We give here some technical facts about \( l \)-adic cohomology, along the lines of standard conjectures stated in [Gro68].

**Proposition 3.3.** Let \( \mathcal{R} \) be a representable functor ...


(Notice here again the use of \texttt{\smallskip} to split two logically distinct parts of the explanations in the “proof”!)

3.C. **Diagrammatic picture of the proof**
The reader will understand better our strategy by looking at the following diagram. At the same time, this provides an example of how to include an EPS or PDF figure. With dvips, one must include encapsulated postscript files (EPS), while pdflatex needs JPG, PNG or (preferably) PDF format (search for Flowchart.pdf in the present source file).
4. Main arguments

The idea is to use a double induction on the weight and coniveau of the involved Hodge structures.

Lemma 4.1. Let $X$ be a nonsingular projective scheme over an algebraically closed field $k$ of characteristic 0. (...)

... ...

5. Further comments

In case the manuscript title is very long (or the list of authors is very long), the header line of odd pages might not have enough space to include this information. You can adjust this by specifying \titlemark and \authormark. Similarly, for titles of sections that would be too long to fit on headers of even pages, use \sectionmark.

The epimath latex style is otherwise pretty standard. The font size and line spacing of the list of references can be adjusted: it is recommended to use \bibliographysize{small} for a more compact presentation. The default heading “References” can be changed, e.g. to “Bibliography”, by setting before \renewcommand{\refname}{Bibliography}. Also, a command like \bibliographymark{References} set before the bibliography list will let ”References” appear in the right page headers instead if the last section – this can be useful if the bibliography list is very long.

Finally, epimath.sty provides special macros \DOIstring{...} to set the DOI on top of the first page, as well as \ARXIV{...}, \HAL{...} and \MR{...} to specify references to the preprint archives arXiv and HAL, and to Math Reviews, respectively, with appropriate hyperlinks. See e.g. [Voi11] and [Voi13] below.
References


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