K. Chandrasekharan (1920 - 2017)

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K. Chandrasekharan, the founder of the School of Mathematics of the Tata Institute of Fundamental Research, passed away on 13 April 2017. A man of intellectual humility and integrity, whose striving for excellence helped the School of Mathematics at TIFR gain the reputation of being a world-class centre for mathematics. He played a major role in the development of mathematics in post-independence India. He did his Ph.D. with K. Ananda Rau, whose advisor was G. H. Hardy. It is due to his efforts that the facsimile edition of the two Ramanujan Notebooks were first published by TIFR.

KC, as he is usually referred to, was born on 21 November 1920 in Bapatla, Guntur district in Andhra Pradesh, which was then a part of the Madras Presidency. His father was a school teacher. KC had his university education at the Presidency College, Madras (now called Chennai). The Madras Presidency comprised the largest part of south India, and Presidency College was a very prestigious institution having excellent scholars in both humanities and sciences on its faculty. For example, amongst the mathematics faculty was Ananda Rau, who was a student of Hardy and a contemporary of Srinivasa Ramanujan during his Cambridge days. In the English department there was K. Swaminathan, who was later the chief editor of the collected works of Mahatma Gandhi, which ran into 90 volumes.

KC was keenly interested both in mathematics and English literature. He once remarked to me with some pride a compliment paid to him by Swaminathan regarding his skills in English language. However, when KC was contemplating the possibilities of pursuing either pure mathematics or English literature at the College, it seems Swaminathan advised him to take up mathematics as he felt that a career in English literature might not hold a good future for him in "independent India". Thus, KC naturally took up mathematics and wrote his Ph.D. thesis with Ananda Rau as his thesis advisor.

KC's mathematical interests centred around analysis and analytic number theory. The most well-known and influential persons in mathematics in Madras at that time were Ananda Rau, R. Vaidyanathaswamy (at the newly created mathematics department in the Madras University) and Fr. Racine at Loyola College. Fr. Racine had received his doctorate under Élie Cartan in Paris. Whereas Ananda Rau represented analytic number theory, the other two brought to KC and other students the awareness of different disciplines. In fact, Vaidyanathaswamy was interested in many areas of mathematics. Indeed, he has the distinction of writing the first textbook on topology in India (which was later published by Chelsea). KC had a very great regard for Vaidyanathaswamy not only for his mathematics but also for his humane qualities. If I correctly recall, KC once told me that he studied mathematical logic with Vaidyanathaswamy. Besides KC, there were other mathematicians of high quality produced by this "Madras School", such as S.S. Pillai, S. Minakshisundaram, Ganapathy Iyer, Kesava Menon and K.G. Ramanathan.

KC went to the Institute for Advanced Study (IAS) at Princeton in 1946. Marshall Stone, who visited Madras around that time, seems to have played a major role in arranging this visit. This also seems to be the case with the visits of Minakshisundaram and Ramanathan to the IAS. I believe that

during his stay in Princeton, KC was Hermann Weyl's assistant. He also got to know John von Neumann very well. During his Princeton years, KC collaborated extensively with Salomon Bochner, who was at Princeton University, which resulted in papers in analysis centering around Fourier transforms. In fact, they wrote a book on Fourier transforms (S. Bochner and K. Chandrasekharan. *Fourier Transforms*. 1950. Princeton University Press. ISBN 9780691095783).

The Tata Institute of Fundamental Research (TIFR) in Bombay (now called Mumbai), was founded in 1945 by Homi Bhabha, a very highly regarded theoretical physicist. TIFR also had D.D. Kosambi and F.W. Levi as its faculty members. The latter was originally a professor at Calcutta University, having come to India from Germany during the Nazi period. After a very short stay at TIFR, Levi returned to Germany to occupy the position he had held before the ascent of the Nazi regime. Kosambi on the other hand was a very versatile scholar who, besides his interest in mathematics—differential geometry, statistics—was a very good Sanskrit scholar and a Marxist historian. He would have indeed fitted very well in any academic institution (such as a good university in the U.S.) which promotes the study of sciences as well as liberal arts. However, he did not seem to have had the personality to be a leader of a group.

Bhabha was looking to recruit very good persons in many fields to TIFR, particularly in mathematics. He visited the Institute for Advanced Study when KC was there, and on the recommendation of Hermann Weyl and von Neumann, Bhabha invited KC to join TIFR. KC accepted this invitation and joined TIFR in 1948. Soon after joining TIFR, KC invited Minakshisundaram to TIFR and their collaboration resulted in a book on "Typical Means" (K. Chandrasekharan and S. Minakshisundaram. *Typical Means*. 1952. Oxford University Press).

KC began to organise mathematical activities at TIFR in a systematic manner. In this task, he seems to have been influenced by what he imbibed during his stay at the IAS. His idea was to build expertise in many important disciplines in mathematics besides his own field of specialisation and to build something similar to a graduate school of a U.S. university so that the students would have to study some basic disciplines before beginning to specialise to work towards a Ph.D. degree. He first offered a position to K.G. Ramanathan who was a student of Carl Ludwig Siegel at IAS and had earned his Ph.D. degree from Princeton University.

Along with Ramanathan, KC initiated such a graduate program at TIFR. A few talented students were selected every year from various parts of India on the basis of interviews. After a year of basic training, they were to attend courses in different fields taught by distinguished mathematicians who were invited to spend a few months at TIFR. The notes of each course were to be written by one or two students who attended them. These lecture notes were written up and published as the "TIFR Lecture Notes Series" and became well-known over the years. Incidentally, KC gave a course on zeta functions that was published in the Lecture Notes Series. The programme of foreign visitors started in the year 1953, the year in which both M.S. Narasimhan and I joined TIFR through the advice of Fr. Racine. That year, Warren Ambrose gave a fairly extensive course ranging over several topics: set topology (covered in three lectures), differentiable manifolds, Lie groups, Hilbert spaces, spectral theorem, Peter–Weyl theorem, etc. Let me name some of the visitors during the initial period of the School of Mathematics: C.L. Siegel, S. Eilenberg, H. Rademacher, H. Mass, M. Eichler, O. Zariski and L. Schwartz. The visit of Laurent Schwartz was of particular importance as he brought to the awareness of the French mathematicians, such as Henri Cartan, the promise shown by the School of Mathematics. Indeed, many French mathematicians visited TIFR later to give courses of lectures. Several talented students joined TIFR and for a number of years the faculty of TIFR consisted of those students who made a mark in their fields of specialisation.

KC also initiated a four-yearly international colloquium on various topics in mathematics and this was supported by the International Mathematical Union (IMU), a practice that continues to this day. The first colloquium was on zeta functions and was held in 1956. Atle Selberg gave four talks in this colloquium and the famous "Selberg trace formula" figured in these talks.

M.S. Narasimhan and I were deputed in 1957 to work in Paris. We were there for three academic

years and returned to TIFR in 1960. This was a great period of mathematics in France. "The Grothendieck Revolution" in algebraic geometry was then the talk of the day in Paris, and since I was studying algebraic geometry with Claude Chevalley, I could get an understanding of Grothendieck's work. I also got to know Jean-Pierre Serre. By the mid-1960s, the work done by the School of Mathematics covered a wide range of topics: algebraic geometry, differential geometry, topology, complex spaces, algebraic and analytic number theory, algebra (particularly centred around the study of projective modules), Lie groups and the study of its discrete groups. Much of these works were of a very high calibre.

TIFR gained the reputation of being a world-class centre for mathematics, a vindication of KC's dedication to excellence. One should mention here that he not only wanted to improve the quality of mathematics at TIFR but also wanted to help the teaching community in India. He had a very good rapport with mathematics teachers at various colleges in Bombay. He was also responsible for creating a training programme for mathematics teachers from colleges all over India. He used to organise annual summer schools at TIFR on various topics in mathematics, where lectures were given by some of the mathematicians from the School of Mathematics. One of KCs important achievements was the setting up of the Department of Mathematics at Bombay University and appointing the renowned statistician Sharadchandra Shankar Shrikhande as the head of the department. He was also responsible for making the University Grants Commission (UGC) recognise the Department of Mathematics of Bombay University as a UGC centre for promoting research. He deputed two members of the School of Mathematics to work with Shrikhande for the development of the centre.

As another significant contribution of KC for the improvement of the state of mathematics in India, one should mention that at the suggestion of Vaidyanathaswamy, who was then the chief editor of the *Journal of the Indian Mathematical Society* (*JIMS*), KC became the chief editor of *JIMS* in 1950 and served in this capacity for eight years until 1958. He was also the chief editor of the *Mathematics Student* for five years from 1953 to 1958. During this period, in an effort to improve the quality of these journals, he encouraged promising Indian mathematicians and invited well-known mathematicians—such as Hermann Weyl and André Weil—to publish their papers in these journals. The proceedings of the first international colloquium held in TIFR was published in *JIMS*.

I still remember that when I met John Coates (mathematician at Cambridge but hailing from Australia) for the first time at a metro station in Paris, the first question he asked me was "How did you manage to do it?" By which he meant how we were able to build a first-rate mathematics centre in India, when at that time there was no comparable institution for mathematics in Australia. When I was at the Institute for Advanced Study in 1975–76, the physicist S. Chandrasekhar, who was also visiting IAS, told me that eminent mathematicians like André Weil expressed a very high opinion of the School of Mathematics at TIFR. One should remark that a similar renaissance in statistics and mathematics was also happening in Calcutta at the Indian Statistical Institute.

KC left TIFR towards the end of 1965 to join ETH in Zurich, Switzerland and remained there till his death. To many of us at TIFR, his decision to leave TIFR came as a surprise since it happened at a time when all his efforts were bearing fruit. Many possible reasons were given for his leaving but one thing that was clear was his rather strained relationship with Homi Bhabha. Though he admired Bhabha, he found faults with many things that Bhabha did. It is indeed sad that TIFR also lost Bhabha very soon in an air crash after KC left. KC was very upset by this event.

After he moved to Zurich, KC became more active in mathematics. In fact, his collaboration with Raghavan Narasimhan, which began at TIFR, continued even after he went to Zurich. He also had many students at ETH. KC had very great administrative skills which were of course very valuable in building up the School of Mathematics. This talent that he had for good administration continued even at ETH, Zurich, especially related to IMU which he served for 20 years in various capacities in its executive committee including as its president during 1971–74.

After I moved to Chennai from TIFR due to personal reasons and was engaged in building the Chennai Mathematical Institute (CMI), I met him a few times at his residence in Zurich. He was

very encouraging of my efforts. In fact, he even wrote letters to influential people to secure financial support for CMI, though unfortunately he did not succeed.

When the centenary year of Vaidyanathaswamy was celebrated, KC wrote an excellent article on him upon my request, which appeared in the magazine *Frontline* (Mathematics, and Beyond: On R. Vaidyanathaswamy. *Frontline*. 13 Jan. 1995: 81–82). He also wrote two books, *A Course on Topological Groups* and *A Course on Integration Theory*, which were published by Hindustan Book Agency.

Whenever I met KC in his residence, his conversation always centred around TIFR. He was always nostalgic about his TIFR days. KC had an intimidating personality and sometimes he could even be rude but behind his rather rough exterior was a person with intellectual humility, integrity and a striving for excellence. He was ever interested in promoting mathematics and extremely encouraging by helping people in bringing out their best.

KC is survived by his wife and two sons.

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Appendix

On the mathematical work of K. Chandrasekharan

R. Balasubramanian

As was the general trend in Madras University in the early 1940s, K. Chandrasekharan's research started with the summability method, in particular Bessel summability, originally developed by S. Minakshisundaram in his Ph.D. thesis. A little later, KC turned towards the study of Fourier series. As he remarks in his paper [C48], "while single Fourier series and to a lesser extent, double Fourier series have been investigated in great detail, multiple Fourier series have not received the same degree of attention." KC also worked on the summation of multiple Fourier series by spherical means, which was initiated by S. Bochner. KCs definition of summation by spherical means is more general than that of Bochner.

From 1956, his interest turned to the zeta function. H. Hamburger had proved that if $f(s) = \sum_n^\infty \frac{a_n}{n^s}$ and $g(s) = \sum_n^\infty \frac{b_n}{n^s}$, convergent in $\Re\mathfrak{e}(s) > 1$ and if $f(s)$ has only finitely many poles, then the equation

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)f(s) = \pi^{-\frac{1-s}{2}}\Gamma\left(\frac{1-s}{2}\right)g(1-s)$$

implies that $f(s) = g(s) =$ constant multiple of $\zeta(s)$. A simple proof of this was given by C.L. Siegel by connecting the *functional equation* to the *modular relation*. Bochner and KC raised the question [BC56]:

$$\text{If } \phi(s) = \sum_n^\infty \frac{a_n}{\lambda_n^s}, \quad \psi(s) = \sum_n^\infty \frac{a_n}{\mu_n^s}, \tag{0.1}$$

then how many independent solutions are there for the equation

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\phi(s) = \pi^{-\frac{\delta-s}{2}}\Gamma\left(\frac{\delta-s}{2}\right)\psi(\delta-s). \tag{0.2}$$

To answer this question, they associate a constant δ_λ with any sequence $\lambda = (\lambda_n)$ and prove an upper bound for the number of solutions in terms of d_λ and d_μ. In particular, if $d_\lambda = d_\mu$ and $\mu_{n+1} - \mu_n = 1$, they prove that $\delta = 1$ or 3 and they tabulate all the solutions in this case. A little later, KC with Mandelbrojt [CM57] discuss the important cases $\delta = 1$ and 3 in detail.

KC's attention then turned to a more general form of functional equations. Let $\Delta = $ product of gamma factors. Consider

$$\Delta(s)\phi(s) = \Delta(\delta - s)\Psi(\delta - s) \tag{0.3}$$

where ϕ, Ψ are as in (0.1).

KC and Raghavan Narasimhan (RN) [CN62] wrote an asymptotic formula for the Riesz means of the coefficient

$$\frac{1}{\Gamma(\rho + 1)} \sum_{n \leq x} a_n (x - \lambda_n)^\rho \tag{0.4}$$

as the main term $M(x)$ plus an error term $E(x)$. Here the main term is given by an integral. They managed to get $O-$result and $\Omega-$result in this generality. They then apply these results to the classical arithmetic functions like $d(n), \sigma(n)$, and $\varphi(n)$; also for the number of integral ideals in an ideal class in a number field and the coefficients of Dedekind Epstein zeta function. They continue their investigation in [CN64] to get L_2 estimate for the error term. This idea has been extensively used by later authors.

For the Riemann zeta function $\zeta(s)$ and its square $(\zeta(s))^2$, G.H. Hardy and J.E. Littlewood had already deduced an approximate functional equation. This was used by them to get an asymptotic formula for L_2 mean and an upper bound for L_4 mean for the Riemann zeta function on the critical line. Realizing the importance of such an approximate functional equation for any Dirichlet series satisfying a functional equation, KC and RN [CN63] deduced an approximate functional equation for any Dirichlet series satisfying a functional equation with multiple gamma factors (0.3) under some mild conditions (like f has at most a polynomial growth in every fixed vertical line). In particular, they apply the results to the Dedekind zeta function and deduce L_2 estimate for the same on the critical line. This idea has been found very useful to study the zeta functions occurring in algebraic number theory.

Another important result of KC and RN is as follows. The functional equation of S. Ramanujan's τ function gives

$$\frac{1}{\Gamma(\rho + 1)} \sum_{n \leq x} \tau(n)(x - n)^\rho = \left(\frac{1}{2\pi}\right)^e \sum_n \left(\frac{x}{n}\right)^{6 + \frac{\rho}{2}} \tau(n) J_{12 + \rho}(4\pi\sqrt{nx}).$$

KC and RN proved that any functional equation will lead to a similar formula and, whats more, they even get it for small value of ρ (where the $J-$function need not be absolutely convergent, but just convergent).

References

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