Hardy-Ramanujan Journal
Vol. 23 (2000) 21-23.

OBITUARY NOTICES

1. PROFESSOR CHEN-JING-RUN (22-05-1933 TO 19-03-1996)

He was one of the finest mathematicians of the Peoples' Republic of China and his death on 19-03-1996 is a serious loss to China. Owing to his important contributions in mathematics he was appointed as a Research Professor of the Institute of Mathematics in 1978 and elected as a Member of Academia Sinica in 1980. Chen was awarded the first rank of National Science Prize, He-Liang-He-Li Prize and Loo-Keng-Hua Mathematics prize. We mention a few of his contributions. We borrow from Acta Mathematica Sinica, new series Vol. 12, No.3 which was kindly supplied to the editors of Hardy-Ramanujan Journal by Professor Jia Chao Hua of Academia Sinica.

A) Sieve Methods and Applications: a) A glorious crowning achievement of these methods was done by Chen who proved that *every sufficiently large even number* $2n$ *can be expressed in the form*

$$2n = p + qr$$

where p, q, r *are odd prime numbers.* (for the history and further details see Acta Mathematica Sinica mentioned above).

b) Chen proved that *for all* $x \geq x_0$ *there exist odd primes* p, q *such that*

$$x \leq pq \leq x + x^{\frac{1}{2}}.$$

B) Waring's Problem: *Chen proved that for every integer* $n \geq 1$ *there exist 37 integers* $x_1 \geq 0, x_2 \geq 0, \ldots, x_{37} \geq 0$ *such that*

$$n = x_1^5 + \ldots + x_{37}^5.$$

C) Lattice Point Problem: We state a contribution of Chen on the Riemann zeta-function.

$$\left|\sum_{n \leq t} n^{-\frac{1}{2}-it}\right| \leq C_\epsilon t^{\frac{6}{37}+\epsilon}$$

where $t \geq 100, \epsilon > 0, C_\epsilon > 0$ *depends only on* ϵ.

D) Least Prime in Arithmetic Progressions: Chen jointly with his student Wang Tianze proved the following: *Let* $1 \leq l < k$. *Then the least prime in*

$$l, l+k, l+2k, l+3k, \ldots$$

is $O(k^{11.5})$ *where the O-constant is absolute.*

E) Goldbach Numbers: An even number which can be expressed as a sum of two primes is called a Goldbach Number. Let $E(x)$ be the number of numbers not exceeding x which are not Goldbach Numbers. Then H.L.Montgomery and R.C.Vaughan proved that *for some* $\delta > 0$ *there holds*

$$E(x) = O(x^{1-\delta})$$

where the O-constant is absolute. Chen showed (jointly with his student Lia Jian-min) that we can take $\delta = \frac{1}{20}$.

F) Exponential Sum: *Let* q *be an integer* $q \geq 2$ *and* $f(x) = a_1x + a_2x^2 + \ldots + a_kx^k$, *where* $k \geq 2$ *and* $a_1, \ldots, a_k$ *are integers with* $(q, a_1, a_2, \ldots, a_k) = 1$. *Then using a theorem of A. Weil, L.K. Hua proved in 1940 that*

$$\left|\sum_{x=1}^{q} \mathrm{Exp}(2\pi i f(x)/q)\right| \leq C(k)x^{1-\frac{1}{k}},$$

Chen showed that we can take

$$C(k) = \begin{cases} Exp(4k) & if k \geq 10 \\ Exp(kA(k)) & if 3 \leq k \leq 9 \end{cases}$$

where $A(3) = 6.1, A(4) = 5.5, A(5) = 5, A(6) = 4.7, A(7) = 4.4, A(8) = 4.2$ *and* $A(9) = 4.05$.

In conclusion the editors wish to express their indebtedness to Professor Jia-Chao-Hua for much help in the preparation of this obituary notice. For further details refer Chen Jingrun: A brief outline of his life and works, by Pan Chengtong and Wang Yuan, Acta Mathematica Sinica, New Series, 1996, Vol. 12, No.3, pp. 225-233.

2. PROFESSOR KARL PRACHAR (- - 1924 to 27-11-1994).

The sad demise of Professor Karl Prachar (the author of the famous treatise PRIMZHALVERTEILUNG) is a serious loss to Analytic Number Theory. A nice result by him is the following theorem.

<u>THEOREM.</u> *Every odd number $N \geq N_0$ can be expressed in the form*

$$N = p_1 + p_2^2 + p_3^3 + p_4^4 + p_5^5$$

where p_1, p_2, p_3, p_4 and p_5 are odd primes.

The editors are indebeted to Professor Krattenthaler of Austria for much help in the preparation of this obituary. For further details see Memories of Karl Prachar by Edmund Hlawka, Vienna, Monatshefte Math. 121 (1996), 1-9.

ACKNOWLEDGEMENT: The editors are thankful to Smt.J.N.Sandhya for much technical help in the preparation of this volume.