

Hardy-Ramanujan Journal
Vol. 24 (2001) 18-19.

ANNOUNCEMENTS

DISTINGUISHED AWARDS

1. Sri AJAI CHAUDHRY has been awarded the Distinguished Award of the Hardy-Ramanujan Society for the following two results.

a) *Every integer m can be expressed as a sum of twelve seventh powers of integers, i.e.*

$$m = \sum_{i=1}^{12} m_i^7 \quad (m, m_i \text{ integers; positive negative or zero})$$

b) *Every rational number R (+ve, -ve or zero) can be expressed as a sum of eight seventh powers of rational numbers R_i (+ve, -ve or zero). i.e.*

$$R = \sum_{i=1}^8 R_i^7.$$

Ref: On sums of seventh powers. Journal of Number Theory 81(2000) no.2, 266-269.

REMARKS. The result a) above is an improvement of a result with fourteen seventh powers due to W.H.J.FUCHS and E.M.WRIGHT, which is more than sixty years old. Sri AJAI CHAUDHRY was the Indian Ambassador to Singapore and later to Beirut. He is now the Indian Ambassador to Brunei.

2) Professor WADIM ZUDILIN has been awarded the Disnguished Award of the Hardy-Ramanujan Society for the following result. *At least one of the four numbers*

$$\zeta(5), \zeta(7), \zeta(9) \text{ and } \zeta(11)$$

is irrational.

Ref: Russian Mathematical Surveys Vol. 56 No.4 (2001) Russian Pages 149-150. The paper of Professor Tanguy Rivoal (mentioned in Hardy-Ramanujan Journal Vol 23, (2000) p.20) will appear in Acta Arithmetica). In the mean while Professor Rivoal has proved that one atleast of the 19 numbers

$$\zeta(5), \zeta(7), \zeta(9), \zeta(11), \zeta(13), \zeta(15), \zeta(17), \zeta(19), \zeta(21)$$

is irrational

SOME RESULTS WHICH DESERVE TO BE WIDELY KNOWN.

1) We define a Goldbach number to be an even number which can be expressed a sum of two odd prime numbers eg. 6,8,10,... It was conjectured by Goldbach that all even numbers from 6 and onwards are Goldbach numbers. We state

THEOREM (HONGZE L1) *The number of even numbers which do not exceed x which are not Goldbach is $O(x^{0.914})$.*

Ref: The exceptional set of Goldbach numbers - II, Acta Arith. 92 (2000), 71-78.

2) THEOREM (JIA, CHAOHUA and LIU, MING-CHIT) *The largest prime factor of*

$$(N+1)(N+2)\dots(N+M)$$

where N and M and positive integers exceeds $N^{1-\frac{1}{26}-\epsilon}$ provided $M = N^{\frac{1}{2}+\epsilon}$. Here $\epsilon > 0$ is arbitrary and $N \geq N_0(\epsilon)$.

Ref: On the largest prime factor of integers, Acta Arithmetica 95(2000) No. 1, p 17-48.

REMARK. For the earlier history see

a) D.R.Heath-Brown and Jia Chaohua, The largest prime factor of integers in an interval-II, J.Reine u.Angew. Math 498(1998), 35-59. (They prove with the exponent $\frac{17}{18}$)

b) G.Harman (unpublished) (exponent $\frac{19}{20}$)

c) J.K.Haugland (Doctoral Thesis, Oxford (1998) exponent $1 - \frac{1}{25}$)

3)THEOREM (ALESSANDRO ZACCAGNINI). *Let*

$$J(x, h) = \int_x^{2x} \left(\pi(t) - \pi(t-h) - \frac{h}{\log t} \right)^2 dt, x \geq h \geq 10.$$

Then with

$$\begin{cases} x^{\frac{1}{6}-\varepsilon(x)} \leq h \leq x, 0 \leq \varepsilon(x) \leq \frac{1}{6} \\ \varepsilon(x) \rightarrow 0 \text{ as } x \rightarrow \infty, \end{cases}$$

we have

$$J(x, h) \ll \frac{xh^2}{(\log x)^2} \left(\varepsilon(x) + \frac{\log \log x}{\log x} \right)^2.$$

Ref: A. Zaccagnini, Primes in almost all short intervals, Acta Arithmetica, LXXXIV, 3(1998), p. 225-244.