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A Biographical Sketch

By C. S. Aravinda and K. Srinivas

S. Srinivasan was an analytic number theorist of repute, with a wide spectrum of interests. His paper [Sri80] 'A remark on Goldbach's Problem', in which he proves certain asymptotic formulas for the Goldbach binary conjecture and the allied twin-primes conjecture, is considered one of his most important works. Given his sensitivity for perfection and precision, he wrote less, but had more, patiently and constantly brewing in his head. He worked for most part of his career as a faculty member in the School of Mathematics of the Tata Institute of Fundamental Research (TIFR) in Mumbai. After retiring in September 2003, he moved to the TIFR Centre in Bangalore, where he rejoined his mentor K. Ramachandra, and worked there until his sad passing away in June 2005.

Early education in Mysore

Born on 8 September 1943 (in Mysore) to an orthodox Hindu family of Iyengars, Srinivasan was the second child to his parents. He had an older brother and a younger sister. His father was V.K.S. Nathachariar and mother Kamalavalli.

Srinivasan was only nine years old when he lost his mother. During those early years of his childhood, Mysore was still the capital city of the erstwhile princely state, ably ruled by the Wodeyar clan. It served as the capital city of their Kingdom of Mysore for nearly six centuries, except for a brief period of interregnum in the 1760s and 70s when Hyder Ali and Tipu Sultan were in power. The Wodeyar kings were patrons of art and culture and contributed significantly to the cultural and economic growth of the city and the state. They were especially known for their love of music and fine arts, and particularly encouraged pursuit of traditional knowledge, while keeping a progressive outlook.

Following India's independence in 1947, Bangalore became the capital of the Mysore State and the administration of the region came under the governance of the elected democracy. When the linguistic reorganization of Indian states took place in 1956, more Kannada-speaking regions came under the new state of Mysore, and the present name of the state 'Karnataka' came into being in 1973. Even long after all these transformations, traces of regality of the city of Mysore still remain, especially with the grand Dasara festival celebration every year with all its component fanfare, giving Mysore its reputation as the cultural capital of Karnataka.

Growing up in the rich cultural ambiance of Mysore must have left a deep impression on Srinivasan's own upbringing. He had a particular liking for classical carnatic music and had fine-tuned his ears to a close and discerning appreciation of the finer nuances of its melodic and rhythmic aspects, all of which must have had some bearing on his strong mathematical inclination. The prevailing air of quietude and peace around also impacted Srinivasan's own personality, as most of his friends and associates remember him as a man of calm and unhurried disposition, and peace.

Srinivasan finished high school from Sarada Vilas High School, one of the best schools of the time. The long and distinguished history of Sarada Vilas Educational Institutions traces all the way back to 1861, when under the directive of the then Mysore King Mummadi Krishnaraja Wodeyar (or Krishnaraja Wodeyar III), one of his chief administrators Rao Bahadur Sri Bhakshi Narasappa initially founded the Sarada Vilas Anglo-Sanskrit School. The High school section, which started in 1917 boasts of noted luminaries such as N. R. Narayana Murthy, founder of the Infosys, a leading establishment of the IT industry, as one of their distinguished alumni. The building where the high school is presently housed, dates back to 1931.

After passing out of his SSLC (Secondary School Leaving Certificate) examination from Sarada Vilas High School in 1957, Srinivasan then shifted to another illustrious institution in Mysore, namely the Yuvaraja's College, for his college studies and obtained his B.Sc. degree in the year 1962.

Making of the Mathematician

Always a keen student of Mathematics, Srinivasan pursued his masters in Mathematics at the University of Mysore, thoughtfully named 'Manasagangotri' – the fountainhead of all intellectual energy – by Kuvempu (pen name of the the celebrated Kannada poet laureate K. V. Puttappa), and earned his M.Sc degree from there in 1964. Former head of the Department of Post Graduate Studies and Research in Mathematics at Manasagangotri, his teacher, S. Bhargava recounts the days when Srinivasan was a student there:

> *Around the time he was a student at the University, I was new to the teaching profession myself. However, I distinctly remember my illustrious senior colleagues Professors K. Venkatachaliengar and T. S. Nanjundiah admiring Srinivasan's approach to learning mathematics and his out-of-the-box type answers in his exams. That meant trouble for him as the answer scripts were to be evaluated by external valuers also. However, we were all happy that he was ultimately mentored by none other than K. Ramachandra.*

Later, reciprocating in equal measure the warmth shown by his teachers at the university during his student days, Srinivasan returned there to spend an extended time, giving a series of lectures on what he had learnt in the meanwhile. As Bhargava says:

> *Several years after his doctoral studies under Ramachandra, Srinivasan visited our department for a few months. Needless to say, some of the students and teachers of the department benefited very much from our interaction with him on sieve methods.*

However, the transit of Srinivasan from Mysore to TIFR, to work with K. Ramachandra, did not happen immediately and was not a direct path. This can also be gleaned from Bhargava's recollection where he says "... we were all happy that he was ultimately mentored by...". From what Ramachandra wrote in [Ra05], Srinivasan worked for the next two years (1964-66) as Demonstrator (rather an unusual terminology) in the Government First Grade College in Hassan, a lively town in the southern part of Karnataka, and is also the district headquarters of the Hassan district.

The city of Hassan enjoys the typical tropical savanna climate all year round and, for this reason, it is often referred to as 'poor man's Ooty'. But the burning desire within himself to pursue his deep interest in Mathematics next lead Srinivasan straight across the country – from the vicinity of the Sahyadri foothills in the south to the proximity of the Himalaya mountains in the north – to Panjab University in Chandigarh, where he joined the Mathematics department as a Research Scholar in 1966.

It was while he was here that he came under the strong influence of Ramachandra's orbit, and followed him. He retained his research scholarship status at Chandigarh, but got his Ph.D. degree in 1969 from University of Bombay (now Mumbai) with Ramachandra as his thesis advisor. He worked for a year, in between, at Allahabad with Hansraj Gupta, a prolific number theorist known for his important work on partitions, before taking up the job as a faculty member in TIFR in 1971.

Some glimpses of Srinivasan's mathematical work

Srinivasan collaborated with several mathematicians that include K. Ramachandra, T. N. Shorey, M. Ram Murty, Hansraj Gupta and R. F. Tichy, and has authored 33 research papers in all as reported in the Mathscinet. Taking cognition of his fastidious approach, we shall not try to discern any one particular theme that he may have pursued for long. In conformity with his constant search for simplicity and penchant for perfection, we shall, therefore, touch upon only a couple of Srinivasan's works as samples to showcase his understanding of some of the deep mathematical works and his attempts to give simple and elegant arguments to arrive at almost the same results.

1. On infinitude of primes

Around 300 BCE, Euclid proved that *there are infinitely many primes*. His arguments are simple, beautiful and novel: Suppose there are only finitely many primes, say $2, 3, 5, \ldots, p$. Then the number $n = 1 + 2 \times 3 \times \cdots \times p$ will have a prime factor other than the finitely many primes listed above!

Since then several proofs of the infinitude primes have been written down, but perhaps none of them really as elegant and economical as that of Euclid. Another proof by Christian Goldbach (in a letter to Euler) is based on Fermat's numbers $F_n = 2^{2^n} + 1, n \geqslant 0$ an integer, is based on the fact that $\gcd(F_n, F_m) = 1$, when $n \neq m$, thereby establishing infinitude of primes.

In [Sri84] makes the following interesting observation: let x_m, $(m \geqslant 1)$, be an increasing sequence of positive integers satisfying

$$x_m \mid x_{m+1}, \quad (x_m, x_{m+1}/x_m) = 1. \tag{1.1}$$

This, as well, immediately implies the infinitude of primes! As an example, $a(n) = n^2 + n + 1$ satisfies $a(n^2) = a(n)a(-n)$ and $(a(n), a(-n)) = 1$, which yields (1.1) for $x_m = a(2^{2^m})$. Further, he shows that for a given integer $r \geqslant 1$, and a prime p, there are infinitely many primes of the form $1 \pmod{p^r}$ by observing

$$(2^{p^{r-1}} - 1, (2^{p^r} - 1)/(2^{p^{r-1}} - 1)) = 1.$$

Furthermore, by elementary arguments, he shows the existence of infinitely many primes $\equiv 1 \pmod{n}$ for all n.

2. On Brun-Titchmarsh Inequality.

Let $\pi(x; k, a)$ denote the number of primes $p \leqslant x$ such that $p \equiv a \pmod{k}$. The Brun-Titchmarsh inequality is the following statement:

$$\pi(x; k, a) \ll \frac{x}{\phi(k) \log \frac{x}{k}}, \tag{2.2}$$

valid uniformly in $k < x$.

In 1980, Shiu [Sh80] generalized the Brun-Titchmarsh problem for a class M of non-negative multiplicative functions f satisfying the conditions:

(i) there exists a positive constant A_1 such that $f(p^l) \leqslant {A_1}^l$ for p prime, and $l \geqslant 1$.

(ii) for every $\varepsilon > 0$, there exists a positive constant $A_2 = A_2(\varepsilon)$ such that $f(n) \leqslant A_2 n^{\varepsilon}$, for $n \geqslant 1$,

and proved the following

Theorem 1. *Let $f \in M$, $0 < \alpha < 1/2$, $0 < \beta < 1/2$ and let a, k be integers satisfying $0 < a < k$, $(a, k) = 1$. Then, as $x \to \infty$,*

$$\sum_{\substack{x-y<n\leq x \\ n\equiv a \pmod{k}}} f(n) \ll \frac{y}{\phi(k)} \frac{1}{\log x} \exp\Big(\sum_{\substack{p\leq x \\ p\nmid k}} \frac{f(p)}{p} \Big). \tag{2.3}$$

uniformly in a, k, and y provided that

$$k < y^{1-\alpha}, \quad x^{\beta} < y \leq x. \tag{2.4}$$

Further, as an application of this theorem, he obtained uniform estimates in short intervals for the functions ${d_r}^l(n), r^\lambda(n)$ and $\delta^\lambda(n)$ in place of $f(n)$ where λ is real number, $d_r(n)$ is the number of ways of writing n as a product of r factors, $r(n)$ is the number of ways of writing n as a sum of two squares and $\delta(n)$ counts the number of square full divisors of n.

In 1997, S. Srinivasan [Sri97a] proved the following weaker version of the above result by using simpler arguments.

Theorem 2. *Let $f \in M, 0 < \alpha(< 1)$ and $0 < \beta(< 1)$. Then as $x \to \infty$, under conditions (i) and (ii) defined above,*

$$\sum_{\substack{x-y<n\leq x \\ n\equiv a \pmod k}} f(n) \ll (a,k)\frac{y}{k}\exp\Big(\sum_{\substack{p\leq y/k \\ (p,k)|(a,k)}} \frac{(f(p))^R}{p}\Big) \tag{2.5}$$

with a certain constant R depending only on α, β, f, uniformly in y and integers a, k provided that

$$1 \leq k < y^{1-\alpha}, \quad x^\beta \leq y \leq x. \tag{2.6}$$

The significance of this result of Srinivasan lies in its simplicity of arguments, yet [quantitatively] implying the results obtained from the stronger Theorem 1.

3. On Hilbert's inequality

One of the main tools employed by Montgomery and Vaughan in deriving the weighted form of large sieve (see [MoVa74]) is the following inequality

$$\left|\sum_{r,s} u_r\bar{u}_s \text{ cosec } \pi(x_r - x_s)\right| \leq c_0 \sum_r |u_r|^2 {\delta_r}^{-1}$$

for any set of complex u_r and a finite set of real x_r distinct modulo 1, where $0 < \delta_r := \min_{s\neq r} \|x_s - x_r\|$ and $c_0 \leqslant 3/2$.

In [Sri97b], Srinivasan gives a simpler proof of the above inequality, albeit with a bigger constant $c_0(< 4.26)$ which turns out to be sufficient for many applications.

Epilogue

After retiring from TIFR Mumbai in September 2003, Srinivasan moved to the Bangalore Centre of TIFR (now TIFR Centre for Applicable Mathematics), where Ramachandra had already moved eight years earlier after his own retirement from TIFR Mumbai.

While in Bangalore, Srinivasan also lent a helping hand to Ramachandra in his efforts to bring his then 25 year old brainchild – the Hardy Ramanujan Journal – to a wider view of the world by making available on the internet the past issues of HRJ which were only available in limited print versions. In particular, from its Volume 1 (1978) to Volume 20 (1997), Srinivasan painstakingly made picture files of each page, and carefully stored the soft copies in the computer. Surely, Ramachandra couldn't have been happier. Although more professional scans of the past issues were redone a few years ago and are now available at https://hrj.episciences.org/browse/volumes, Srinivasan's labour of love is still visible at https://www.imsc.res.in/~balu/Hrj/book/ for everyone to see!

Now, one can only conjecture as to what might have been the extent of Srinivasan's possible contribution to HRJ, if it were not for his sudden demise on 1st June 2005 while on a pilgrimage tour to the north with his wife. Yes, Srinivasan was married to Rajalaxmi on 13th July 1983, at a relatively late age when he was close to 40; always the ones preferring to remain away from public gaze, Srinivasan and his wife had a quiet and peaceful life.

It was during the time Srinivasan spent in TIFR Mumbai that much of his mathematical creativity flowered – the soil was fertile, the sky was open and the monsoon rains showered. Reading the last paragraph of Ram Murty's article in this volume, reminiscing his association with Srinivasan, especially the line "It is only the written word that survives, which is but a faint shadow of what we have thought, of what we were and of what we have learned", one cannot help remembering the following two lines from Fitzgerald's *Rubaiyat of Omar Khayyam*:

That every Hyacinth the Garden wears
Dropt in its Lap from some once lovely Head.

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