Photographs of Professor Ivan Matveevich VINOGRADOV (14-09-1891 to 21-03-1983) on his ninetieth birthday.

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Professor K.Ramachandra
The President,
Hardy-Ramanujan Society

Moscow, February 26, 1980

Dear Professor Ramachandra,

Thank you for your kind letter of December 22, 1979, inviting me to accept the fellowship in the Hardy-Ramanujan Society. I accept it with pleasure and want to express my best wishes to the newly founded society which is named after the mathematicians whose work had a great influence on the number theory in this century.

With best regards,

Sincerely yours,

I.M.Vinogradov

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EDITORIAL

Thirty years ago when we started the HARDY-RAMANUJAN SOCIETY we requested Professor Ivan Matveevich VINOGRADOV (14-09-1891 to 21-03-1983) to be an honorary fellow of the Hardy-Ramanujan Society. He blessed us and encouraged us and accepted to be an honorary fellow of the Society. We have been publishing HARDY-RAMANUJAN JOURNAL since 22-12-1978. (The publications of the first volume was delayed due to unavoidable difficulties and could not be published before 22-12-1978). Thanks to the blessings of Professor IVAN MATVEEVICH VINOGRADOV. We have been successful in publishing one volume every year on December 22 (Ramanujan's Birthday). It may be recalled here that the revolutionary result

$$|\zeta(\sigma+it)| \ \leq \ C_1 \ (t^{(1-\sigma)^{3/2})})^{C_2} \ (\log t)^{2/3} \ (t \geq 100, \frac{1}{2} \leq \sigma \leq 1) \tag{1}$$

is practically due to Professor IVAN MATVEEVICH VINOGRADOV. (C_1, C_2 are some positive absolute constants) (see: K.Ramachandra and A.Sankaranarayanan, A remark on VINOGRADOV'S mean value theorem, The J. of Analysis 3(1995)111-129). Here as well as elsewhere

$$\zeta(s) = \sum_{n=1}^{\infty}(n^{-s} - \int_{n}^{n+1} \frac{du}{u^s}) + \frac{1}{s-1} \ (s = \sigma + it) \tag{2}$$

which coincides with $\sum_{n=1}^{\infty} n^{-s}$ in $\sigma > 1$.

VINOGRADOV's result gives rise to

$$\zeta(s) \neq 0 \text{ in } \sigma \geq 1 - \mathrm{c}(\log \mathrm{t})^{-\frac{2}{3}}(\log\log \mathrm{t})^{-\frac{1}{3}} \ (\mathrm{t} \geq 100) \tag{3}$$

(For economical constants c, C_1, C_2 see a paper by KEVIN FORD*) where c is a positive absolute constant. (Of course the result $\zeta(1+it) \neq 0$ is due to J.HADAMARD, and the method of getting (3)from (1) is due to EDMUND LANDAU (See for example the book

"Riemann zeta-function written by K.RAMACHANDRA, published by Ramanujan Institute in 1979)). Professor K.RAMACHANDRA had the honour to meet the great mathematician VINOGRADOV once in 1971 and a second time in 1981.

*[K.F.] KEVIN FORD, VINOGRADOV's integral and bounds for the Riemann zeta-function, Proc. London Math.Soc. 3(85)(2002)565-633.

Note that for all $t \geq 100, \frac{1}{2} \leq \sigma \leq 1$ the estimate

$$\begin{aligned} |\zeta(\sigma+it)| &\leq \sum_{n=1}^{[t]} \left(\frac{1}{n^\sigma} + \int_n^{n+1} \frac{du}{u^\sigma} \right) + 1 \\ &\leq \sum_{n=1}^{[t]} (n^{-\sigma} + n^{-\sigma}) \ + 1 \\ &\leq 2t^{1-\sigma} \sum_{n=1}^{[t]} n^{-1} \ + 1 \\ &\leq 2^{t^{1-\sigma}} \int_1^{t+1} n^{-1} \ + 1 \leq \ 2t^{1-\sigma} \ \log(t+1) + 1, \end{aligned}$$

is trivial. The revolutionary result (1) due to I.M.VINOGRADOV is an extraordinary improvement (of this estimate) which is extremely useful in many arithmetical applications. Also I.M.VINOGRADOV has great results in additive number theory (Waring's problem and additive prime number theory).

EDITORS